Mechanism of Self-regulation in a Simple Model of Hierarchically Organized Market

V. V. Gafiychuk
Institute for Applied Problems of Mathematics and Mechanics, National Academy of Science of Ukraine 3b, Naukova St. L'viv, 290601, Ukraine.

I. A. Lubashevsky, Yu. L. Klimontovich
Moscow State University, Department of Physics, Vorobievy Gory, Moscow 119899, Russia.
(February 5, 2008)

We propose a model for a market which structure is of the tree form. Each branch of the tree is composed by identical firms, its root (the branch of the first level) is formed by the firms producing raw material, and the branches of the last level are the retail outlets. The branching points (tree nodes) are micromarkets for firms forming branches connected with a given node. The prices and the production rate are controlled by the ballance in supply and demand, the competition is assumed to be perfect. We show that such a market functions perfectly: the prices are specified by the production expense only, whereas demand determines the production rate.

We construct an efficiency functional which extremal gives the governing equations for the market. It turns out that this ideal market is degenerated with respect to its structure. It is shown, that such market functions ideally: the prices are determined by the costs on production of the goods, and the level of production of the goods of any kind defined only by demand for the goods of this sort.

I. INTRODUCTION

The characteristic properties of the commodity market evolution have been intensively studied during recent years. However in the present time theoretical description of self-organization phenomena in microeconomic market is still a practicaly unresolved challenging problem. The reason is that the commodity market is a radically open system, i.e. such a system, which exists while and if it exchanges by energy, substance and, that is of not less importance, by information with the surrounding world (with the social and physical environment). This is the main difference between market systems and physical systems. Really, for physical systems it is possible to indicate a state of a thermodynamic equilibrium, where all streams are equal to zero. Such equilibrium state is usually described with small amount of macroscopic parameters - parameters of "order". But economic systems exist while they have the streams of commodity and money. Besides, the number of parameters, which define their state at first glance, seems to be actually endless.

Nevertheless, we can consider far from equilibrium dissipative systems found in physics and chemistry as some rough analogs of such economic systems from the viewpoint of common features of evolution. During recent twenty years these dissipative systems were profoundly investigated (see, for example, [1–5]). It has been found, that their processes of dissipation can produce the self-organization in various types of dissipative structures.

It should be noted, that physical or chemical system at a moment of the origin of dissipative structures transforms into a state with an intermediate degree of chaosity [4]. The entropy of this state is less maximum, corresponding to thermodynamic equilibrium. At the same time it is possible to describe the state with a set of macroscopic (or mesoscopic) parameters of order. Due to the slow change of these parameters the evolution of such systems is usually realized as the process of transition through the sequence of nonequilibrium quasistationary states, distinguished from each other by their dissipative structures.

In economic systems also appear the complex dissipative structures stipulated by unaware will of people. This structures arise from individual interaction of a plenty of agents and spontaneous formation of some order in their relationships (for the review of this problem from the viewpoint of a synergetics, see, for example, [6,7]).

As an example of such self-organization phenomena one can consider the creation and function of trade networks in the systems with distributed auctions, investigated in papers [8–12]. It had been supposed, that the number of trade agents is beforehand given and would not vary in time. It should be noted that self-organization and evolution of these structures is similar to nonequilibrium phase transitions under the changes of external parameters [12].

In the first stage of the development the self-organization theory in microeconomics it is necessary to choose some adequate zero approximation, which actually is a detailed description of the market ideal state - its “norm”. Such norm is well-known. It is an equilibrium market, where the interests of buyers and sellers completely coincide with each other in that way, that within the given price the value of supply is equal to the value of demand. However, for the analysis of self-organization processes more detailed description is required, for example, to pair interactions of sellers and buyers.

Apparently, such rather detailed description of economics as a whole does not have much sense, because the markets of radically different products would be slightly associated. For example, the market of meat items and the market of furniture can interact between themselves mainly through changes in financial states of the whole collection of the consumers. It should be noted, that such situation is similar to the situation, realized in living organisms. Indeed, if a living organism is not in extreme conditions, each of its organs is supplied with amount of blood, completely satisfying its needs, regardless of the other organ's function. It is supplied by the system of large arteries. These arteries form the infinite tank of blood for system of separate organs, and the heart replies to some aggregated information about the state of organism as a whole [13].

Therefore it would be worthwhile to restrict our consideration to any commodity market, related from the viewpoint of possibility of their mutual substitution. Producers, dealers and consumers of these goods will form a common connected network, carrying on the goods from the producers of raw material up to the buyers which buy these goods in the retail shops.

Nevertheless, the set of products, made on the given market, form only a small part of the whole public market. The state of the rest of the economic system has to be taken into account in some set of governing parameters, which actually do not influence the local changes on the given market and reflect aggregated information about society as a whole.

In other words, for the analysis of self-organization processes it seems reasonable to restrict ourselves to research of some small markets of goods, which nevertheless form a common connected system between consumers and all producers of these goods. Such mesoscopic market (mesomarket) is a subject of research in the present paper. Concerning the common requirements applied to the possible model of its norm, it is worhty to single out the following items.

First, it should be of an intermediate degree of chaosity. In other words , on the one hand, individual behavior of the participants of ideal market is not prescribed from any center. More likely, it is determined by their own goals, based on the small part of information about the market state. On the other hand, the ideal market contains structures, generated by individual interaction of the participants as a result of the establishment of spontaneous order in their ratios. These structures form the streams of goods and corresponding flow of money in the opposite direction.

These streams of money bear some kind of information self-processing, which provides a way for each participant to react adequately on various changes of market state, basing only on a small share of information, which aggregates the information about market state as a whole (see [14–16]).

It should be noted, that strictly administrative (completely ordered) system, based on the ideal overall plan as well as formally equivalent to it Walras' "auctioneer" (completely unregulated system, where everyone can contact each other through some process such as "tatonment") cannot function in reality, because for proper functioning these systems require physically infinite time to collect the information [17,18].

Summing up this item we can formulate, that a model of norm has to contain individual interaction of the participants, described with classical laws of demand - supply. And the realization of their contacts has to be given by the micromarket network structure, where physically finite amount of the participants comes into contact.

Second the prices, arising in the result of individual trade relations, normally have to be determined only by real material costs on commodity production. It should be noted, however, that this correspondence should be not an outcome of the prices control, but a result of self-organization processes.

Third, spontaneously arising structures should be organized hierarchically. This is due to the fact of the existence of huge amount of various goods in the market in contrast to small amount of raw materials. Therefore, before the final goods will be originated , material, of which they are made, will be processed during the stages of their production for many times. Their sequence and content are determined by the sort of the produced product and are various for different types of products. The simple example of such hierarchical structure is the structure of the tree form. The root of this tree is formed by firms, obtaining raw materials, and the branches of the last level are the retail shops.

Fourth, the ideal market should be characterized by separability concerning the production of the commodity. In other words, let $\{\alpha\}$ is a collection of the final types of goods of the given market, $\boldsymbol{p} = \{p_\alpha\}$ is their price in retail shops and $\boldsymbol{S}(\boldsymbol{p}, \phi) = \{S_\alpha(\boldsymbol{p}, \phi)\}$ - their demand function, i.e. a consumption level at a given prices. The demand function depends also on some internal parameters ϕ, describing the state of consumers, for example, their average yearly income and so forth. Then the change in the demand for the goods of sort α (i.e. change of the function $S_\alpha(\boldsymbol{p}, \phi)$, for example, as a result of variations in ϕ) parameters should not affect prices and the level of production

of the other types of goods $\alpha' \neq \alpha$ under their invariable demand (i.e. in the absence of changes in the function $S_{\alpha'}(\boldsymbol{p}, \phi)$).

Really, in an ideal case the consumption level of the goods of sort α and their price p_α are the result of the balance of total costs and utility of their production. Therefore, if the given balance is disturbed only because of a change in demand for the goods of the other sort $\alpha' \neq \alpha$ without technology change, then it points to an imperfection of the market and can result in chaos in the commodity production. The same is true for one sort of goods, if customers derive various groups, separated from each other in space so, that they are supplied with various market elements.

It should be noted, that the given requirement of separability of the ideal market is a typical condition, imposed on the process of commodity production and on properties of separate producers one with another interaction. This requirement does not impose any restrictions on possible immediate interrelations in demand of the consumers for the goods of various sort. Such interrelations can arise, for example, because of one individual is a consumer of different types of goods .

In the present paper we have formulated a simple model of the commodity market, which functions ideally. In this case we consider, that the structure of the market has a form of a tree, and competition is perfect.

II. MODEL

Let us now consider the market of some goods, made of one sort of raw material. Collection of the consumers $\mathcal{M}$ of these goods we shall present as a union

$$\mathcal{M} = \bigcup_{\alpha} \boldsymbol{m}_\alpha \tag{2.1}$$

of various groups $\boldsymbol{m}_\alpha$. The difference of these groups is defined in such a way that they consume the different sort of goods, or that they acquire formally the common sort of goods, but are supplied by different branches of the market structure.

It should be noted, that in the first case groups $\boldsymbol{m}_\alpha$ and $\boldsymbol{m}_{\alpha'}$ (for $\alpha \neq \alpha'$) can be physically derived by the same collection of people. In the second case - different consumers are separated either on space, or belonging to various strata of society.

Let us suppose, that the structure of the given market $\mathcal{N}$, formed with industrial and commercial network, has a form of a tree (Fig. 1). The branch i of this tree is a collection of n_i independent firms, bringing out the products of sort i, which buy the products from firms, belonging to the lower level of the network hierarchy $\mathcal{N}$, and sell the results of their activity to the firms of higher level. Firms, forming the common branch i, are considered as identical from the viewpoint of technological process and their trade relations. The root of the tree $\mathcal{N}$ (the branch of the first level) is formed by the firms, obtaining and processing raw material. The tree branches of the last level are the "points" of retail trade, supplying only one group of consumers. Besides we suppose, that for any branch i the number of firms n_i, belonging to this branch is a large value

$$n_i \gg 1 \qquad \forall i. \tag{2.2}$$

The equation (2.2) allows us to consider the values $\{n_i\}$ as continuous variable $(n_i + 1 \approx n_i)$.

Each network node $\mathcal{N}$, for example, node $\mathcal{B}$ is a micromarket, in which only those firms, which belong to the branch $\{i^{\mathcal{B}}_{in}\}$, participate going in a node $\mathcal{B}$, and firms, belonging to branches $\{i^{\mathcal{B}}_{out}\}$, going out of the given node (sellers and buyers in this market, respectively, Fig. 2). All products on the given branch are sold under one price $p_{\mathcal{B}}$. Firms, which belong to the root of a tree $\mathcal{N}$, extracting raw material, and the firms of the last level of a hierarchy sell the goods directly to the consumers.

The network $\mathcal{N}$ actually determines the economic ratios between the market participants and sets interrelations of product flows. We shall consider the situations, when the market of the considered goods exists, i. e. all the streams should be more than zero.

Let us measure the level of the firm production, which belongs to the branch i in a unites of the initial raw material flow x_i, "flowing through" the given firm. It should be noted, that in this system of unites the value x having dimensionality $(\textit{material}\,)/(\textit{time})$, and index i really determines the sort of production of the considered firm. The full products stream of $X_i$can be represented

$$X_i = n_i x_i. \tag{2.3}$$

Due to the material conservation law, in network nodes $\mathcal{N}$ relations

$$X_{i_{in}^{\mathcal{B}}} = \sum_{j\in\{i_{out}^{\mathcal{B}}\}} X_j. \tag{2.4}$$

are fulfilled for any node $\mathcal{B}$.

The whole production of the last level firms is acquired by the consumers. In doing so, for a branch i_α, supplying a group of the consumers $\boldsymbol{m}_\alpha$ we can write the following:

$$X_{i_\alpha} = X_\alpha, \tag{2.5}$$

where X_α – consumption level of the goods by group $\boldsymbol{m}_\alpha$, expressed in raw material flow unites.

It should be noted, that produced goods on this mesomarket, can include not only the raw material but also other additional materials. The latest, however, are acquired by firms individually and their costs are included into the cost of production. The raw material of the considered model is like a binding of the different producers in the single network, that is expressed in the conservation laws (2.4).

The trade interaction on the micromarket $\mathcal{B}$ results in the origin of interchanged production price. Taking into account the preceding, we shall measure the interchanged production by the effective price $p_{\mathcal{B}}$ of the raw material unit. Then in the result of its activity each firm, which belongs to the branch i, obtains the unit time profit π_i, which is equal to:

$$\pi_i = (p_i^{(s)} - p_i^{(b)})x_i - t_i(x_i). \tag{2.6}$$

Here $p_i^{(s)}$ and $p_i^{(b)}$ are prices on the micromarkets, where firms i represent itself as the seller and buyer, respectively, $t_i(x)$ – it's total costs for production with x_i level. Following the standard view, we suppose, that $t_i(x)$ is increasing convex function, growing faster than $x^{1+\epsilon}$ (where ϵ - some positive constant). Besides, we consider that

$$t_i(0) > 0 \qquad \forall i. \tag{2.7}$$

It should be noted, that the nonequality (2.7) is a condition, imposed on the specific production costs, i.e. production costs, referred to one firm, considered as indivisible. Total costs T_i, connected with the activity of all firms, which form the branch i, are equal $T_i = n_i t(x_i)$ and depend on two arguments: n_i and x_i. If the value x_i is considered as the given parameter of production, then by virtue of (2.3) we can express $T_i = X_i t_i(x_i)/x_i$ and therefore $T_i \to 0$ at $X_i \to 0$, since the condition (2.2) allows to consider n_i as a continuous variable.

For firms, obtaining raw material we have the expression

$$p_i^{(b)}\Big|_{i\in root} = 0. \tag{2.8}$$

We suppose, that the individual goal of each firm is reaching the maximum profit. In this case the strategy of the firm i we can describe as the productions with the x_i level, satisfying the following condition

$$\frac{\partial \pi_i(x_i)}{\partial x_i} = 0. \tag{2.9}$$

The change in the demand, in particular, causes the change in the goods production level, and therefore, results in the value of gained profit. The latter, in its turn, stimulates or suppresses activity of the firms. In the same case this stipulates the arising or vanishing of the firms in the market. If a competition is perfect (i.e. when there are no barriers for the entrance of new firms into the existing market) at the equilibrium status, the profit received by the firm, taking into account all costs, should be equal to zero (see, for example, [19,20]). Assuming this condition executed, the value x_i should satisfy the conditions:

$$\pi_i(x_i) = 0. \tag{2.10}$$

The consumers, purchasing the goods in the "points" of retail trade, try to maximize their utility. Taking into account their utility and budget constrains we shall describe the behavior of the consumers group $\boldsymbol{m}_\alpha$ by a positive definite demand function $S_\alpha(\boldsymbol{p}, \phi)$

$$X_\alpha = S_\alpha(\boldsymbol{p}, \phi), \tag{2.11}$$

where $\boldsymbol{p} = \{p_\alpha\}$ - collection of prices of all final goods in the shops of retail trade, and the parameters ϕ reflect total expenditure. The equations (2.3) - (2.5) and (2.8) - (2.11) describe the functioning of the given market and, in particular, determine the amount of firms $\{n_i\}$, participating in production and trade of commodities.

Let us analyze characteristics of the formulated model.

III. IDEAL SELF-REGULATION

At the given structure of the micromarkets the unknown variables are: level of separate firms production$\{x_i\}$, their number $\{n_i\}$ in different branches and prices on the micromarkets $\{p_{\mathcal{B}}\}$.

The expression (2.6) for the profit π_i of the firm i contains only two independent variables, the difference $(p_i^{(s)}-p_i^{(b)})$ and the level of production x_i. Therefore, on the accepted assumptions about type of functions $t_i(x)$ the system of two equations (2.9), (2.10) has a single solution x_i^*, being the root of the equation

$$\left.\frac{d\ln t_i(x)}{d\ln x}\right|_{x=x_i^*}=1. \tag{3.1}$$

To the rate of production x_i^* corresponds the difference of prices

$$\Delta p_i \overset{\text{def}}{=} (p_i^{(s)}-p_i^{(b)})=t_i'(x_i^*). \tag{3.2}$$

The outcome (3.2) allows immediately to find the price $p_{\mathcal{B}}$, established on the micromarket (node) $\mathcal{B}$. Really, considering the condition (2.8) and summing up expression (3.2) along the path $\mathcal{P}_{\mathcal{B}}$, leading from the root of the tree $\mathcal{N}$ to the given node $\mathcal{B}$, (Fig. 1) we have the following expression for the price

$$p_{\mathcal{B}}=\sum_{i\in\mathcal{P}_{\mathcal{B}}} t_i'(x_i^*). \tag{3.3}$$

Selecting, in particular, micromarket of the last level α, i.e. the node, linking the firms of the last level and the group of the consumers $\boldsymbol{m}_\alpha$, we find the prices for goods of sort α

$$f_\alpha\equiv p_\alpha=\sum_{i\in\mathcal{P}_\alpha} t_i'(x_i^*). \tag{3.4}$$

Thus we arrive to the following conclusion:

Suggestion 1 *In the given model with the fixed structure $\mathcal{N}$ the prices $\{p_{\mathcal{B}}\}$, established on the micromarkets $\mathcal{N}$, are determined only by technological process, and not by demand of the consumers.*

This is a direct consequence of a perfect competition. In other words, at the perfect competition, the prices reflect only the costs of the goods production in spite of the fact that physically they are established as the result of supply and demand balance. It should be noted, that this result is similar to that, received in Leontjev models.

Full stream of production X_i, “flowing” through the branch i, reasonably depends on demand. Thereby, in the given model just an amount of firms $\{n_i\}$, forming the branches $\{i\}$ of the market structure $\mathcal{N}$ is controlled by demand of the consumers $\mathcal{M}$. In other words, the change of values $\{n_i\}$ is a manifestation of self-regulationts of this type of market.

As seen from conditions (2.5) and (2.11), total stream of the goods $X_\alpha=X_{i_\alpha}$, made by firms of the last level, which form, for example, the branch α, is determined by

$$X_\alpha=S_\alpha(\boldsymbol{f},\phi), \tag{3.5}$$

where $\boldsymbol{f}=\{f_\alpha\}$.

The conservation laws (2.4)of the material streams on the micromarkets for an arbitrary branch i allow us to write an expression:

$$X_i=\sum_{\alpha\in\mathcal{M}_i} X_\alpha \tag{3.6}$$

where $\mathcal{M}_i$ is a collection of groups of the consumers, connected directly with the given branch i through the higher than the branch i hierarchy level branches (Fig. 1).

$$n_i=\frac{1}{x_i^*}\sum_{\alpha\in\mathcal{M}_i} S_\alpha(\boldsymbol{f},\phi). \tag{3.7}$$

In the conclusion to this part of paper, it should be noted, that the equation (3.5) testifies about separability of the given model of the market from the viewpoint of the absence of mutual influence of the consumers on different types of commodity. In other words

Suggestion 2 *Any kind of changes in demands of one group of the consumers $\boldsymbol{m}_\alpha$ for the goods of sort α in any way does not affect the price and production of the other sort of goods $\alpha' \neq \alpha$.*

IV. VARIATIONAL PRINCIPLE FOR THE IDEAL MARKET MODEL

The set of values $\{x_i\}$, $\{n_i\}$ and $\{p_{\mathcal{B}}\}$ actually gives all main conditions of the production and trade on the given commodities market. In other words these parameters are parameters "of order". The strategy of firms and individual behavior of the consumers (correlation(2.9), (2.10) and (2.11)) determines concrete values of these parameters. Nonetheless, the formulated model of the market contains an implicit image of one more parameter of the order, namely, structure of the local micromarkets, which defines the order of firms trade interactions. Really, with organization of the commodity market of the given type, initially fixed is the multitude of the consumers $\mathcal{M}$ and the source of raw material. Realization of technological process and commercial network is a concrete solutions of the problem on supplying consumers with the required goods. Solution of the given problem, basically, can be ambiguous and in this case market self-organization process has to select this realization of the network $\mathcal{N}$.

Next paragraph is devoted to the analysis of the ideal market model from this point of view.

A. Global function of consumption utility

At the beginning we shall construct some function $\mathcal{U}(\boldsymbol{X})$ from $\boldsymbol{X} = \{X_\alpha\}$, extremum properties of which define the function of demand of the consumers $\boldsymbol{S}(\boldsymbol{f}, \phi) = \{S_\alpha(\boldsymbol{f}, \phi)\}$. Let us suppose that collection of the consumers $\mathcal{M}$ consists of $\{\mathcal{M}_k\}$ individual or various groups, considered as separated individuals

$$\mathcal{M} = \bigcup_k \mathcal{M}_k. \tag{4.1}$$

A full set of goods, necessary for the life activity of each of them, is $\{\gamma\} = \{\alpha\} \bigcup \{\beta\}$, where, as was used earlier, $\{\alpha\}$ are goods, delivered on the considered market and $\{\beta\}$ are goods, made on other markets. In this part of the paper we shall suppose that appropriate prices $\boldsymbol{c} = (\boldsymbol{p}, \boldsymbol{q})$ for the goods $\{\gamma\}$ are given. Here, following the previous paragraphs, we used the denotations $\boldsymbol{c} = \{c_\gamma\}$, $\boldsymbol{p} = \{p_\alpha\}$ and $\boldsymbol{q} = \{q_\beta\}$.

Individuals $\{\mathcal{M}_k\}$ are independent. Each individual k on gaining a set of the goods $\boldsymbol{Z}^{(k)} = (\boldsymbol{X}^{(k)}, \boldsymbol{Y}^{(k)})$ (where $\boldsymbol{Z}^{(k)} = \{Z_\gamma^{(k)}\}$, $\boldsymbol{X}^{(k)} = \{X_\alpha^{(k)}\}$ and $\boldsymbol{Y}^{(k)} = \{Y_\beta^{(k)}\}$) maximize his utility function $U_k(\boldsymbol{Z}^{(k)}) = U_k(\boldsymbol{X}^{(k)}, \boldsymbol{Y}^{(k)})$ proceeding from the budget constraint ϕ_k. In other words each individual solves the problem

$$\begin{gathered} \max_{\left(\boldsymbol{X}^{(k)}, \boldsymbol{Y}^{(k)}\right)} U_k(\boldsymbol{X}^{(k)}, \boldsymbol{Y}^{(k)}) \\ \text{if} \quad (\boldsymbol{X}^{(k)} \cdot \boldsymbol{p} + \boldsymbol{Y}^{(k)} \cdot \boldsymbol{q}) = \phi_k \end{gathered} \tag{4.2}$$

A full consumption level of the goods type α is given by expression

$$X_\alpha = \sum_k X_\alpha^{(k)} \tag{4.3}$$

The problem (4.2) is reduced to solution of the system of equations

$$\frac{\partial U_k(\boldsymbol{Z}^{(k)})}{\partial Z_\gamma^{(k)}} = \Lambda_k c_\gamma, \tag{4.4}$$

where $\Lambda_k > 0$ are the Lagrange multipliers, the index γ runs through all the set $\{\gamma\}$. The solution of the given system of equations determines the demand function of the k consumer

$$\boldsymbol{Z}^{(k)} = \boldsymbol{S}^{(k)}(\boldsymbol{c}\Lambda_k) = (\{S_\alpha^{(k)}(\boldsymbol{c}\Lambda_k)\}, \{S_\beta^{(k)}(\boldsymbol{c}\Lambda_k)\}), \tag{4.5}$$

which depends on both the prices $\boldsymbol{c}$, and the value Λ_k, the latter itself becoming the function of the prices $\boldsymbol{c}$ and of the budget constraints ϕ_k, and satisfies the equation

$$\boldsymbol{S}^{(k)}[\boldsymbol{c}\Lambda_k(\boldsymbol{c},\phi_k)]\cdot\boldsymbol{c}=\phi_k. \tag{4.6}$$

The utility function $U_k(\boldsymbol{Z}^{(k)})$ is defined with the accuracy of monotone transformation $U_k\to\mathcal{G}(U_k)$, that conjugates also with transformation $\Lambda_k\to\Lambda_k/\mathcal{G}'(U_k)$. The final type of the function $S^k(c,\phi_k)$ remains constant. The dependence (4.5) as the argument contains the product $\boldsymbol{c}\Lambda_k(\boldsymbol{c},\phi_k)$, where the second multiplier (in his turn, possibly, multiplied on the function of $1/\mathcal{G}'(U_k)$ type) actually aggregates in itself an information about the state of the market of commodities γ as a whole and of budget constrains of the consumer k. Evident dependence in this type of function $S_\gamma^{(k)}$ on the price $c_{\gamma'}$ with $\gamma\neq\gamma'$ reflects mutual influence of the goods of sorts γ and γ' for the process of their choice. In a considered model a set of goods α in its own right comply with some requirement of the person, while other goods $\{\beta\}$correspond to other independent requirements. Therefor, it is natural to expect, that a choice of the set X^k will be defined only by prices $\{p\}$ for these goods and by common budget constraint, aggregated in some uniform multiplicand depending already from all prices $\boldsymbol{c}$ and constrains ϕ_k. Let's note, that the given assumption is similar to the aggregated description of the utility function of various commodity groups [21–23], however it has the difference (see. a chapter IV B). Such behavior of the consumers will be realized, if using some transformation $\mathcal{G}$ the utility function U_k can be reduced to such type, that for any α and β

$$\frac{\partial^2 U_k(\boldsymbol{X}^{(k)},\boldsymbol{Y}^{(k)})}{\partial X_\alpha^{(k)}\partial Y_\beta^{(k)}}=0. \tag{4.7}$$

Taking into account a correlation (4.7) we shall accept the following assumption.

Suggestion 3 *We suppose, that for the goods $\{\alpha\}$ of the ideal mesoscopic market for any consumer k there is a utility function $U_k(\boldsymbol{Z}^{(k)})$, representable as,*

$$U_k(\boldsymbol{X}^{(k)},\boldsymbol{Y}^{(k)})=u_k(\boldsymbol{X}^{(k)})+v_k(\boldsymbol{Y}^{(k)}). \tag{4.8}$$

Then, by virtue of (4.4) - (4.6), demand function for the goods $\boldsymbol{Z}^{(k)}=(\boldsymbol{X}^{(k)},\boldsymbol{Y}^{(k)})$ can be rewritten as

$$\boldsymbol{S}^{(k)}(\boldsymbol{c},\phi_k)=(\boldsymbol{S}_x^{(k)}(\boldsymbol{p}\Lambda_k),\boldsymbol{S}_y^{(k)}(\boldsymbol{q}\Lambda_k)), \tag{4.9}$$

where functions $\boldsymbol{S}_x^{(k)}(\boldsymbol{p}\Lambda_k)=\{S_\alpha^{(k)}(\boldsymbol{p}\Lambda_k)\}$, $\boldsymbol{S}_y^{(k)}(\boldsymbol{q}\Lambda_k)=\{S_\beta^{(k)}(\boldsymbol{q}\Lambda_k)\}$, satisfy the conditions

$$\left.\frac{\partial u_k(\boldsymbol{X}^{(k)})}{\partial X_\alpha^{(k)}}\right|_{\boldsymbol{X}^{(k)}=\boldsymbol{S}_x^{(k)}}=\Lambda_k p_\alpha, \tag{4.10a}$$

$$\left.\frac{\partial v_k(\boldsymbol{Y}^{(k)})}{\partial Y_\beta^{(k)}}\right|_{\boldsymbol{Y}^{(k)}=\boldsymbol{S}_y^{(k)}}=\Lambda_k q_\beta, \tag{4.10b}$$

And the function $\Lambda_k(\boldsymbol{p},\boldsymbol{q},\phi_k)$ is found from the equation

$$\boldsymbol{S}_x^{(k)}(\boldsymbol{p}\Lambda_k)\cdot\boldsymbol{p}+\boldsymbol{S}_y^{(k)}(\boldsymbol{q}\Lambda_k)\cdot\boldsymbol{q}=\phi_k. \tag{4.11}$$

The value of the Λ_k parameter is controlled by a state of whole economic system. Therefore local changes of the prices $\boldsymbol{p}$ on mesoscopic market of the goods $\{\alpha\}$ should vary the value Λ_k. Let's demonstrate it on the example of small fluctuations of the prices $\delta\boldsymbol{c}=(\delta\boldsymbol{p},\delta\boldsymbol{q})$. Varying expression (4.11) under condition $\phi_k=const$, we get

$$\sum_\alpha(1-\epsilon_\alpha^{(k)})S_\alpha^{(k)}\delta p_\alpha+\sum_\beta(1-\epsilon_\beta^{(k)})S_\beta^{(k)}\delta q_\beta=(\sum_\alpha\epsilon_\alpha^{(k)}S_\alpha^{(k)}p_\alpha+\sum_\beta\epsilon_\beta^{(k)}S_\beta^{(k)}p_\beta)\frac{\delta\Lambda_k}{\Lambda_k}, \tag{4.12}$$

where

$$\epsilon_\gamma=-\left.\frac{\partial S_\gamma^{(k)}}{\partial c_\gamma}\right|_{\Lambda_k=const} \tag{4.13}$$

From this immediately follows, that, when the number N of independent mesomarkets is rather great and the fluctuations of the prices is not significant, so, they do not change the state of the consumer, the mean value $\langle \delta\Lambda_k/\Lambda_k \rangle \to 0$, at $N \to \infty$ for $1/\sqrt{N}$. This leads us to

Suggestion 4 *At the local changes on the market of the goods $\{\alpha\}$ the value Λ_k can be considered as some constant macroscopic variable, describing common state of the consumer k*

Taking into account relation (4.10a), from this assertion we also receive, that

Suggestion 5 *At the local changes on the market of the goods $\{\alpha\}$ the strategy of the consumer k can be described as finding of a maxima of the following function*

$$\max_{\boldsymbol{X}^{(k)}} \left[\frac{1}{\Lambda_k} u^{(k)}(\boldsymbol{X}^{(k)}) - \boldsymbol{p} \cdot \boldsymbol{X}^{(k)} \right].$$

In this case function

$$\frac{1}{\Lambda_k} u^{(k)}(\boldsymbol{X}^{(k)})$$

can be considered as a utility function of the goods required$\boldsymbol{X}^{(k)}$, depending also from some macroscopic parameters. This utility function not only defines the preference of a choice, but also measures this preference in monetary units.

Let′s now consider the result of collective behavior of individuals $\{\mathcal{M}_k\}$. A full consumption level X_α of the goods of type α by virtue of (4.3) is

$$X_\alpha = \sum_k S_\alpha^{(k)}(\boldsymbol{p}\Lambda_k). \tag{4.14}$$

Let′s define the function

$$\mathcal{U}(\boldsymbol{p}, \{\Lambda_k\}) \stackrel{\text{def}}{=} \sum_k \frac{1}{\Lambda_k} u_k(\boldsymbol{S}_x^{(k)}(\boldsymbol{p}\Lambda_k)). \tag{4.15}$$

If in these two expressions (4.14), (4.15) we consider values $\boldsymbol{p}$ as some formal set of parameters, than expression (4.14), can be inverted, resulted in some dependence, specifying the vector $\boldsymbol{p}$ as the function of a vector $\boldsymbol{X}$. The latter allows us to consider expression (4.15) as the function $\mathcal{U}(\boldsymbol{X}|\{\Lambda_k\})$ of arguments $\boldsymbol{X}$. By virtue of (4.10a),(4.10b),(4.14),(4.15) we get

$$\frac{\partial \mathcal{U}}{\partial X_\alpha} = \sum \frac{\partial \mathcal{U}}{\partial p_{\alpha'}} \frac{\partial p_{\alpha'}}{\partial X_\alpha} = \sum_{k,\alpha',\alpha''} \frac{\partial \mathcal{U}}{\Lambda_k \partial S_{\alpha''}} \frac{\partial S_{\alpha''}}{\partial p_{\alpha'}} \frac{\partial p_{\alpha'}}{\partial X_\alpha} = \sum_{k,\alpha',\alpha''} p_{\alpha''} \frac{\partial S_{\alpha''}}{\partial p_{\alpha'}} \frac{\partial p_{\alpha'}}{\partial X_\alpha}$$

$$= \sum_{\alpha',\alpha''} p_{\alpha''} \frac{\partial p_{\alpha'}}{\partial X_\alpha} \frac{\partial}{\partial p_{\alpha'}} \left(\sum_k S_{\alpha''}^{(k)} \right) = \sum_{\alpha',\alpha''} p_{\alpha''} \frac{\partial p_{\alpha'}}{\partial X_\alpha} \frac{\partial X_{\alpha''}}{\partial p_{\alpha'}} = \sum_{\alpha''} p_{\alpha''} \delta_{\alpha,\alpha''} = p_\alpha$$

Thus, collective behavior of the consumers on the market of the goods $\{\alpha\}$ can be presented as optimization of the global utility function of consumption $\mathcal{U}(\boldsymbol{p}, \{\Lambda_k\})$, which depends also on some macroscopic parameters $\{\Lambda_k\}$and is measured in monetary units. In other words:

Suggestion 6 *At the local changes on the market of the goods $\{\alpha\}$ the strategy of cooperative behavior of the consumers can be described as finding a maximum of the following function:*

$$\max_{\boldsymbol{X}} \left[\mathcal{U}(\boldsymbol{X}|\{\Lambda\}) - \boldsymbol{p} \cdot \boldsymbol{X} \right]. \tag{4.16}$$

The common function of demand $\boldsymbol{S}(\boldsymbol{p}|\{\Lambda\})$ of the goods $\{\alpha\}$ satisfies the equation

$$\left. \frac{\partial \mathcal{U}(\boldsymbol{X}|\{\Lambda\})}{\partial X_\alpha} \right|_{\boldsymbol{X}=\boldsymbol{S}} = p_\alpha. \tag{4.17}$$

Measurability of the global utility function in monetary units allows to compare it with costs of the goods production on the given market and to describe its operation as a whole in terms of the variational problem. Next section is devoted to consideration of this problem

B. Functional of the market efficiency

Let's use the actually known description of the market system in terms of global efficiency criterion. Let′s set a functional of efficiency $\mathcal{D}$ of the market goods $\{\alpha\}$ to be:

$$\mathcal{D} = \mathcal{U}(\{X_\alpha\}|\{\Lambda\}) - \sum_i n_i t_i(x_i), \tag{4.18}$$

Here indexes α and i run over all units of sets $\mathcal{M}$ and $\mathcal{N}$, respectively. The arguments of this functional are $\{n_i\}, \{x_i\}, X_\alpha$ and the structure of the network of supply $\mathcal{N}$, which gives conservation laws of material streams, and, hence, the interrelation between values $\{n_i\}, \{x_i\}$, and X_α values. The first term in the right part of expression (4.18) is a global utility of the goods consumption with the level X_α, and the second term - the production costs .

Suggestion 7 *The laws of functioning of the ideal mesomarket can be rewritten as the equations for the extremals of (4.18) functional, considering the network $\mathcal{N}$, as given.*

Indeed, variables $\{n_i\}, \{x_i\}, X_\alpha$ are not independent, and are connected by relations (2.4) and (2.5). Let's take advantage of the Lagrange method, which in this case will correspond to the prices $p_{\mathcal{B}}$ on the micromarket at node $\mathcal{B}$

$$\mathcal{D}^{\mathcal{P}} = \mathcal{D} + \sum_\alpha p_\alpha (n_{i_\alpha} x_{i_\alpha} - X_\alpha) + \sum_{\mathcal{B}} p_{\mathcal{B}} (n_j x_j|_{j=i^{\mathcal{B}}_{in}} - \sum_{j\in\{i^{\mathcal{B}}_{out}\}} n_j x_j). \tag{4.19}$$

Here index $\mathcal{B}$ runs over whole nodes (micromarkets) of the network $\mathcal{N}$ except nodes, connecting immediately firms and consumers, and $\{p_\alpha\}$ and $\{p_{\mathcal{B}}\}$ are Lagrange multipliers (prices) attributed by the nodes network $\mathcal{N}$. For functional (4.19) arguments $\{n_i\}$, $\{x_i\}$, $\{X_\alpha\}$ and $\{p_\alpha\}$ and $\{p_{\mathcal{B}}\}$ are supposed to be independent. As it is well-known, in this case the extremals of functionals (4.18) and (4.19) coincide with each other. Relation (4.19) can be also rewritten as:

$$\mathcal{D}^{\mathcal{P}} = \mathcal{U}(\{X_\alpha\}|\{\Lambda\}) - \sum_\alpha p_\alpha X_\alpha + \sum_i n_i \pi_i(x_i\ ,\ p_i^{(s)} - p_i^{(b)}), \tag{4.20}$$

where the function $\pi_i(\ldots\ ,\ \ldots)$ is determined by the formula (2.6). Then differentiating the expression (4.19) with respect to variables $\{p_\alpha\}$ and $\{p_{\mathcal{B}}\}$, and expression (4.20) with respect to variables $\{n_i\}$, $\{x_i\}$, $\{X_\alpha\}$ we get the equations of the model, formulated in the part II, where the Lagrange multipliers act as prices, and the demand function $\boldsymbol{X} = \boldsymbol{S}(\boldsymbol{p}|\{\Lambda\})$ satisfies the criterion (4.17).

Let′s now consider the condition of extremality of a functional (4.19)(or(4.20))concerning all the parameters of the order, including the parameters describing the production and trade system. As follows from the result of part II, the expression of the functional $\mathcal{D}^{\mathcal{P}}_*$, received after maximization over all arguments, except a structure of the network $\mathcal{N}$, is determined next formula

$$\mathcal{D}^{\mathcal{P}}_* = \mathcal{U}(\{X_\alpha\}|\{\Lambda\}) - \sum_\alpha f_\alpha X_\alpha. \tag{4.21}$$

Values $\{f_\alpha\}$, and therefore values $\{X_\alpha\}$, are given by production process and geometry of the network $\mathcal{N}$. If their changes are such that variations $\{\delta f_\alpha\}$, and $\{\delta X_\alpha\}$ are small ,then, by virtue of (4.17) and (4.21) the variation of functional $\mathcal{D}^{\mathcal{P}}_*$ is:

$$\delta \mathcal{D}^{\mathcal{P}}_* \approx - \sum_\alpha X_\alpha \delta f_\alpha. \tag{4.22}$$

From (4.22) it follows the comparison criterion of the function efficiency of ideal markets, identical for their nomenclature of the goods of consumption $\{\alpha\}$, which differ by organization of the goods production. Namely:

Suggestion 8 *Increasing the value of functional efficiency of ideal mesomarket, owing to the changes of its production and trade structure corresponds to decreasing, in the average, retail prices for the goods of consumption.*

The given result allows, at least on an intuitive level to hypothesize that for ideal mesomarket the functional of efficiency will have the extremum including about market structures. Really, it seems to be quite reasonable, that due to self-organization process only those structures will survive, which offer the goods with lower prices to the consumers.

The second result, following from the formula(4.22), is, that for the ideal market exists a plenty of realization of production and trade structure, which are identical from the viewpoint of extremality of functional efficiency. In other words:

Suggestion 9 *The ideal market is degenerates concerning the network of supply $\mathcal{N}$.*

Really, according to expression (3.4) the amount of parameters, specifying the value of $\{f_\alpha\}$, far exceeds the number of these values in consequence of hierarchical organization of the network $\mathcal{N}$. That is why the set of values $\{f_\alpha\}$ can be obtained by a large amount of methods. In particular, two realizations of the network $\mathcal{N}$ will result in the single value $\mathcal{D}_*^{\mathcal{P}}$, if they contain a single amount of hierarchy levels. Firms, belonging to one level are characterized by the same function of the costs $t_i(x_i)$, and networks are differed only by organization of intermediate bifurcations (Fig.3). In other words, market structures with identical technology, which have various organization of the micromarket network, are equivalent on the ideal market.

V. REMARKS

In the present paper we tried to demonstrate, that the problem of self-regulation of market systems can be solved through the appearance of production and trade hierarchical structures. The classical economic theory and Walras′s auctioneer suggestion assumes the availability of the whole information about an economic state of the market. However, it is physically impossible due to the large number of the participants of the market system. This limitation can be removed, if the market participants in their contacts restrict to a small amount of other participants, so that to form a connected micromarkets network. In doing so, however, occurs another problem.

Each market participant, contacting with the limited number of others, receives a small share of information, on which he orients himself planning and selecting the strategy of his activity. At first glance such limited information (usually as the prices for some production of the last range) immediately cannot say him, what and how much to produce. Really, at availability of the developed system of the micromarkets the result of his work, usually is an intermediate product (semiproducts). And the market participant is involved in a long production chain, connecting the production of raw materials and the final goods offered to the consumers. And the prices are established during the process of mutual convention of a small amount of people.

The formulated model of the ideal market with hierarchical structures of the micromarkets demonstrated the existence of self-regulation process in such systems. It allows the market participants to adequately react on changes of the consumers demand, basing only on a value of the prices on the two appropriate micromarkets, connected together by the participants. The conservation laws of material flows and streams of money on the micromarkets are the basis for this self-regulation . Owing to the fact that the stream of money "flows" in back direction (from the consumers to the firms, obtaining raw materials) in contrast to the stream of materials, the smaller-sized stream of money are going into all bigger-sized streams. The integration in money streams provide that the production prices aggregate information about the state and the demand of the consumers on more and more large scales. The latter provides the self-processing of information, latent in relation of the prices on semiproducts of different hierarchical levels.

In summary we shall note the following.

First, in an accepted model it was considered, that production and trade network $\mathcal{N}$ has a form of a tree. For the market with a perfect competition such assumption is quite justified. Really, as follows from (2.9), (2.10), the conclusion about the independence between the Δp_i difference in prices and demand is not connected with network $\mathcal{N}$ geometry, and is the sequence of the perfect competition. Let assume now, that to some micromarket (node of the network $\mathcal{N}$) has two incoming branches (technological ways), connecting it with firms, obtaining raw material (root of the network $\mathcal{N}$). Then the prices for production, proposed on the given micromarket for these two routes of production, should be different (except accident agreement). Therefore one of these routes of production has to disappear. And the network $\mathcal{N}$ has to become of a tree form (graph without cycles).

Second, for the ideal market the prices the amount of firms participating in production and trade, and also the streams of goods appear to be defined values. The market deviation from ideality, for example, when the competition is not perfect, and the firms producing the same production are not identical, will produce the change in these values. However, if these deviations are not too large, it is possible to expect, that also the changes in the prices and in the commodity production level also will be insignificant. The situation varies significantly, concerning market structures. Owing to a degeneracy of market norm over production and trade structures, small deviations from perfectness can result in, that during self-organizing will "survive" the structures of various form. Upon it small variations of parameters unideality can stipulate significant changes in the market structures. It is likely that, this effect was observed in paper [12]. It should be noted, that a plenty of realizations of production and trade structure, which is identical from the viewpoint of ideal market, points to the possibility of existence of any macroscopic parameters of order, characterizing an architecture of these structures in aggregated form.

In summary to the given part of the paper we note that the assumption about possibilities of reduction of the common utility function (4.7) is similar to aggregated description of utility function for the goods, belonging to different groups [21–23]. The main difference is, that in the proposed approach, budget constrains are not imposed individually on the goods of each mesomarket and are considered as common constraints for *all* sorts of the goods. As a result, in the given approach individual indexes of the prices are absent. The description of the local behavior of the consumers on mesomarket (condition (4.16)) in its form is different from the global optimality condition (4.2) . Besides the utility function in the form (4.7) is uniquely defined, and that doesn′t allow beforehand the homogeneity of the first degree.

We thank to Prof. I. G. Pospelov and Dr. S. Guriev for the useful remarks and recommendations. Paper also was supported in parts by U1I000 and U1I200 grants from International Science Fundation, Funds of Fundamental Researches of Russia Federation and Ukraine.

[1] Nikolis G. , Prigogin I. *Self-organization in nonequilibrum systems. From dissipativ structures to the order through fluctuations* (J.Wiley and Sons, 1977).

[2] Haken H. *Synergetics. An introduction. Springer Ser.in Synergetics. v.1* (Springer-verlag, 1988).

[3] Haken H. *Information and self-organizing. A macroscopic approach to complex systems* (Springer-verlag, 1988).

[4] Klimontovich Yu. L. *Statistical theory of open systems* (Janus, Moscow, 1995).

[5] Kerner B. S. , Osipov V. V. *Autosolitons. A new approach to problem of self-organozation and turbulance* (Kluwer, 1994).

[6] Hayek F. A. *The fatal conceint. The errors of socialism* (The Univ. Chicago press, 1988).

[7] Weidlich W. *Physics and social science – the approach of synergetics* // Phys. Rep. 1991, v. 204, n. 1, pp. 1–163.

[8] Baesemann R. C. *The formation of small market places in a competitive economic process – the dynamics of agglomeration* // Econometrica, 1977, v. 45, n. 2, pp. 361–376.

[9] Rubinstein A. , Wolinsky A. *Equilibrium in a market with sequential bargaining* // Econometrica 1985, v. 53, n. 5, pp. 1133–1150.

[10] Wolinsky A. *Information revelatio in a market with pairwise meetings* // Econometrica 1990, v. 58, n. 1, pp. 1–24.

[11] van Raalte C. L. , Gilles R. P. *Endogeneous Formation of Trade Center: An Evolutionary Approach* (Department of Economics and CentER, Tilburg University, 1994).

[12] Guriev S. ,Shahova M. *Model of self-organizing of commercial networks in economy with incomplete infrastructure* In: Mathematical models of the analysis of data and management in microeconomics (Moscow, 1995).

[13] Mchedeshvili G. *Microcirculation of blood: common features of regulation and violations* (Science, Leningrad, 1989).

[14] Lubashevsky I. A. , Gafiychuk V. V. *A simple model of self-regulation in large naturla systems* // J. Envir. Syst. 1995, v. 23, n. 3, pp. 281–289.

[15] Gafiychuk V. V. , Lubashevsky I. A. *On hierarchical Structures Arising Spontaneously in Markets with Perfect Competition* //J. Env. Systems. - v. 25, No. 2. - p. 159-166. (1996-1997).

[16] Lubashevsky I. A. , Gafiychuk V. V. *Cooperative mechanism of self-regulation in hierarchical living systems* e- print. http://xxx. lanl. gov/abs/adap-org/9808003 (1998), submitted to SIAM J. Appl. Math.

[17] Pospelov I. G. *Dynamic Model of the Market* // Economy and Math. Methods, 1988, v. 24, no. 3.

[18] Pospelov I. G. *Optimal strategy of behaviour in the dynamic market models* // An Automation and telemechanics, 1989, no.2, pp. 113 - 123.

[19] B. R. Binger and E. Hoffman. *Microeconomics with Calculus* (Harper Collins Publishers, 1988).

[20] Scherer F. M. , Ross D. *Industrial Market Structure and Economic Performance* (Houghton Mifflin Company, Boston, 1990).

[21] Shananin A. A. *A Condition of integrability in the problem of finite goods* // Rep. AS USSR, 1987, v. 294, no. 3, pp. 553 - 555.

[22] Vratenkov S. D. ,Shananin A. A. *Analysis of a structure of consumer demand with economic indexes* (Computer Center AS USSR, Moscow, 1991).

[23] Shananin A. A. *A nonparametric method of the analysis of the consumer's demand structure* // Math. Simulation, 1993, v. 5, no. 9, pp. 3 - 17.

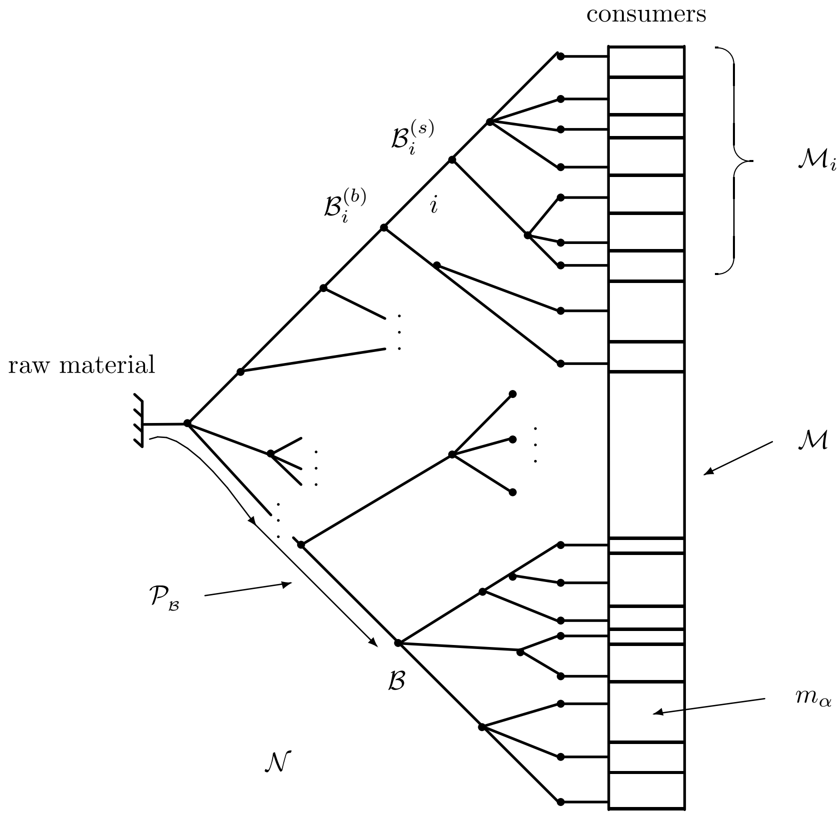

FIG. 1. Structure of the mezomarket under consideration.

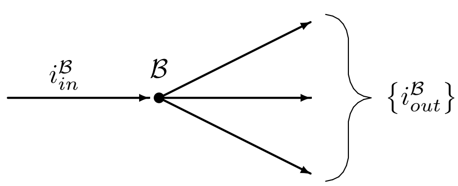

FIG. 2. Structure of an elementary micromarket.

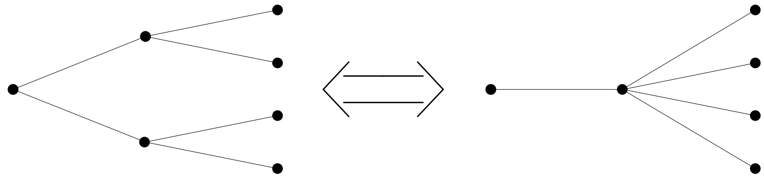

FIG. 3. Equivalent fragments of the production-trade network.